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Three Tank System Control Using Neuro - Fuzzy Model Predictive Control

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ABSTRACT
Three-tank (3T) system is the most representative didactical equipment used as a bench mark system for system modeling, identification and control. A real target representing 3T system has been used for generating data that is used for developing a linear model based on auto-regressive exogenous (ARX) method, and neuro-fuzzy (NF) network technique. The developed models have been used as an internal model of the model predictive control (MPC). Control actions of the NFMPC algorithm has been determined using nonlinear programming methods based on sequential quadratic programming (SQP) technique. The developed NFMPC algorithm has shown good performance and set point tracking over the linear MPC algorithm based on ARX model for controlling the 3T system.

Keywords
Three-tank system, model predictive control (MPC), auto regressive exogenous (ARX) model, neuro-fuzzy model, sequential quadratic programming (SQP).

1. INTRODUCTION
Liquid level control has a very large application domain in industry, three-tank (3T) system is the most representative didactical equipment widely used as a benchmark system for system modeling, identification, control, fault detection and diagnosis, as well as for fault-tolerant control. The system exhibits typical characteristics of a strong nonlinearity with different possibilities of disturbance, which makes the system useful to serve as a test environment for algorithms concerning state estimation, parameter identification, and control of hybrid systems.

The three-tank liquid level control system is a multi-input-multi-output (MIMO) system, but with the valves (actuators) between the tanks closed, each tank can be treated as a single-input-single-output system (SISO). The typical challenging control issue involved in the system is the tuning of the level controllers to keep the desired liquid level in each tank in the presence of disturbances and extreme variations in the process dynamics and tuning settings. Our goal is to analyze the efficiency of a neuro-fuzzy model predictive control (NFMPC) on 3T control level system.

There have been a lot of good articles that deals with the three tank system problem, Popescu and Mastorakis proposed an optimal tuning of PI controller for the adjustment of the 3T system based on hardware and software knowledge and adjustment [1], Hao *et al*. adapted a Fuzzy adaptive Smith predictive control system that is composed of Smith predictor and fuzzy adaptive controller [2], the algorithm uses fuzzy adaptive PID control to improve the resistance ability to random disturbance and Smith predictive control to overcome the time-delay character of the 3T controlled object, Kovacs, *et al*. proposed an optimal control method based on H_2/H_∞ [3], and Abdelkader *et al*. have designed a multiple observer using the principle of interpolation of local observers and implement it on 3T system under the condition of unknown inputs [4].

Fuzzy set theory originated by Zadeh [5], was originally developed to quantitatively and effectively handle problems involving uncertainty, ambiguity and vagueness, and provides a useful technique for dealing with complex nonlinear systems faced in the real world. Li and Hu [6] used a combination of fuzzy Takego-Sugeno dynamic model with Riccati equation as an adaptive fault control scheme for level control of the 3T system. Pham and Li proposed a fuzzy inverse reasoning controller using fuzzy relational equation to deduce control actions appropriate for the desired the 3T system output [7]. Suresh *et al*. applied a fuzzy controller to the 3T system to overcome the problem associated with design and analysis of traditional feedback control systems which are based on mathematical models [8].

Design of a fuzzy logic controller is accompanied with certain problems regarding design of membership functions (type and number of membership functions, their shape and range, etc.), and choosing appropriate fuzzy rules. Moreover, developing a rule base is one of the most time-consuming parts of designing a fuzzy logic controller. Usually it is very difficult to transform human knowledge and experience into a rule base of fuzzy logic controller. Frequently, designing a fuzzy logic controller requires a number of trial and error iterations, and even then, it is very difficult to ensure that the designed controller is an optimal one. Hence there is a need for developing efficient methods to tune membership functions, i.e. to obtain optimal shapes, ranges and number of membership functions, etc. and to obtain optimal rule base. A neuro-fuzzy modeling technique recently developed overcomes all the problems previously mentioned. Neuro-fuzzy technique is based on determining the parameters of fuzzy models using optimization algorithms developed in neural network training. There have been some good articles dealing with the problem of nonlinearity using neuro-fuzzy technique and applied it to different industrial processes [9 – 12]. Moreover, the application of neuro-fuzzy technique to 3T system has been appointed by Hu and Rose [13], they proposed a generalized predictive controller (GPC) using a radial basis function (RBF) based neuro-fuzzy model to perform the online multi-step prediction and the controller design. Mok and Chan [14] proposed a fault detection and isolation scheme based on fuzzy rules constructed from neuro-fuzzy network that models the residual of the 3T system.

In this research, we are aiming on developing a neuro-fuzzy and ARX models of the 3T system based on the data generated for a real target. The developed models are incorporated in the model predictive control (MPC) algorithm.

2. NEURO-FUZZY IDENTIFICATION OF DYNAMICAL SYSTEMS

A key issue to address when designing a nonlinear model predictive control (NMPC) controller is the choice of process model, and the type of model structure of the nonlinear process model to be used. There are models based on fundamental relationship which are very complex or not available at all. There are models based on empirical data such as Voltera Kernels, Fourier series, wavelets, RBF and multilayer perceptron (MLP) that have shown to provide useful unified presentation for a wide class of non-linear systems for more information about nonlinear models utilized for NMPC see [15]. The use of fuzzy systems for nonlinear identification is not motivated only by their approximation capabilities but also by their capacity to extract linguistic information in the form of IF-THEN rules which typically describe compact sets.

The task of model identification can be mainly concentrated on the estimation of rules, the distribution of fuzzy sets and the centers of the membership function, the adjustability of the mentioned parameters allow the fuzzy model to adapt to the addressed process. Because fuzzy logic and NN encode the information in a parallel and distributed architecture in a numerical framework, hence it is possible to convert fuzzy logic architecture to a NN in order to adjust these parameters in the fuzzy model. This method of adjusting fuzzy model parameters using the training algorithms in NN literature is defined as neuro-fuzzy identification.

To build a neuro-fuzzy model of dynamic system based on the input-output data [16]. The output of the dynamic system at time t is $y(t)$ and the input $u(t)$. The data set will be described as

$$Z^t = \{y(1), u(1), \ldots., y(t), u(t)\} \quad (1)$$

Mapping from past data Z^{t-1} to the next output $y(t)$ is called the estimated output of the predictor model

$$\hat{y}(t) = f_{FUZ}(Z^{t-1}) \quad (2)$$

The essence of identification using fuzzy systems is to tray represent the function f_{FUZ} by means of a fuzzy model. The form of a fuzzy system is a parameterizable mapping is

$$\hat{y}(t|\theta) = f_{FUZ}(Z^{t-1}|\theta) \quad (3)$$

Where, θ is the vector of parameters to be chosen (position and shape of the membership function, consequence of the rules, etc.). The choice of these parameters is guided by the information embedded in the data. The structure of equation 2 is a very general structure and it has a drawback that the data set is continuously increasing, for this reason, it is better to use a vector of fixed dimension. So the general model will have the following form.

$$\hat{y}(t|\theta) = f_{FUZ}(\varphi(t)|\theta) \quad (4)$$

The vector φ is known as the regression vector and which takes the form of the nonlinear auto-regressive exogenous (NARX) model structure with time delay t_d and the order of the polynomial n_y and n_u.

$$\varphi = y(t-1), \ldots \ldots, y(t-n_y), u(t-t_d), \ldots \ldots u(t-t_d-n_u) \quad (5)$$

Using this parameterization, our problem is concentrated on choosing regressors in $\varphi(t)$, finding the structure of fuzzy system $f(.,.)$ and finding the parameters θ of the fuzzy system. The regressors are chosen in a way that can be implemented in our MPC algorithm, the fuzzy model structure and parameters finding were obtained using Fuzzy logic Toolbox in MatLab environment.

3. NEURO-FUZZY MODEL PREDICTIVE CONTROL (NFMPC)

Model predictive control (MPC) scheme (Figure 1) is based on the receding horizon control approach, which can be summarized by the following steps:

- Predict the system output over the range of future times.
- Assume that the desired outputs are known.
- Choose a set of future control, which minimizes the future errors between the predicted future output and the future desired output.
- Use the first element of future control moves as a current input, and repeat the whole process at the next instant.

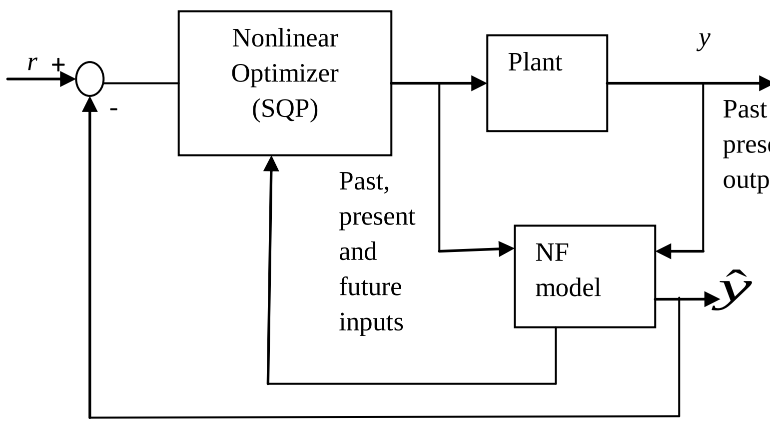


Figure 1. Neuro-fuzzy model predictive control structure

The first step in receding horizon control is to predict the system over the range of future times. This could be done by using a one-step ahead predictor Equation 6. The k-step ahead prediction of the system output can be calculated by shifting the expression forward in time while substituting predictions for actual measurements where these do not exist Equation 6.

$$\begin{aligned}\hat{y}(t+k) = f_{nn}[&\hat{y}(t+k-1),\dots\dots\dots,\hat{y}(t+k-\min[k,n_y]),\\ &y(t-1),\dots\dots\dots\dots\dots\dots,y(t-\max[n_y-k,0]),\\ &u(t+k-t_d),\dots\dots\dots\dots,u(t+k-t_d-n_u)]\end{aligned} \quad (6)$$

It is assumed that the observation of the output is available up to time *t-1* only; for this reason, the output $\hat{y}(t)$ of the neuro-fuzzy network model enters the expression instead of the real output *y(t)*.

The objective function is the sum of square errors of the residuals between predicted outputs and the set point values over the prediction horizon; a term penalizing the rate of change of the manipulated variable is often included as well. Mathematically, the NFMPC problem can be stated in vector form as follows:

$$\begin{aligned} J(t,U(t)) &= \Gamma^{y}[R(t+1)-\hat{Y}(t+1)]^{T}[R(t+1)-\hat{Y}(t+1)]+\Gamma^{u}\Delta U^{T}(t)\Delta U(t) \\ &= \Gamma^{y}E^{T}(t+1)E(t+1)+\Gamma^{u}\Delta U^{T}(t)\Delta U(t) \end{aligned} \tag{7}$$

where

$$R(t+1) = [r(t+1), \ldots, r(t+p)]^{T}$$
$$E(t+1) = [e(t+1/t), \ldots, e(t+p/t)]^{T}$$
$$\hat{Y}(t+1) = [\hat{y}(t+1/t), \ldots, \hat{y}(t+p/t)]^{T}$$

$$\Delta U(t) = [\Delta u(t), \ldots, \Delta u(t+m-1)]^{T}$$

$$e(t+k/t) = r(t+k) - \hat{y}(t+k/t) \qquad \text{for } k = 1, \ldots, p$$

subjected to,

$$\begin{aligned} y_{low} &\le \hat{y} \le y_{high} \\ u_{low} &\le u \le u_{high} \end{aligned} \tag{8}$$

where *m* is the control horizon, *p* is the prediction horizon, Γ^{u} and Γ^{y} are the weighting coefficients matrices, $\hat{Y}(t+1)$ is the predicted output vector, *R(t+1)* is the set point vector, and *ΔU(t)* is the rate of change of the manipulated variable *(Δu(t) = u(t)-u(t-1))*.

4. THREE TANK LABORATORY TEST-BED

The tested-bed plant figure 2 consists of three tanks that can be filled with two identical, independent pumps acting on the tank 1 and 2. The pumps deliver the liquid flows Q_1 and Q_2 and they can be continuously manipulated from a flow of *0* to a maximum flow Q_{max}. The follows in the pipes between tank *1* 1nd tank *2* and the third tank are manipulated using switched valves V_1 and V_3. These interaction can be seen from the following mathematical model of the three tank (3T) system [17]:

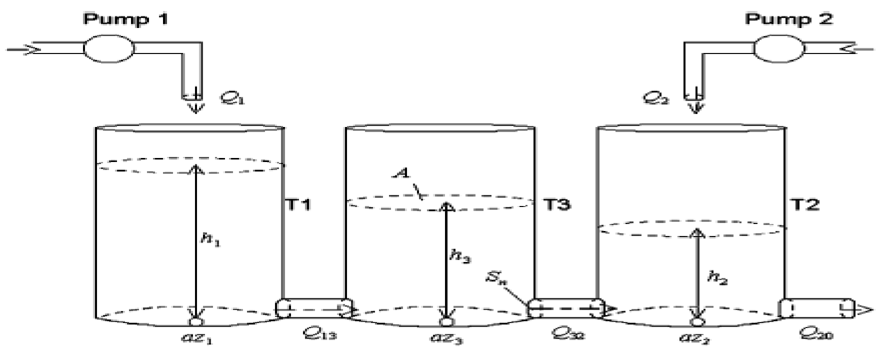


Figure 2. Three tank system

$$\frac{dh_1}{dt} = \frac{1}{A}(Q_1 - Q_{13} - Q_{1leak}) \tag{9}$$

$$\frac{dh_3}{dt} = \frac{1}{A}(Q_{13} - Q_{32} - Q_{3leak}) \tag{10}$$

$$\frac{dh_2}{dt} = \frac{1}{A}(Q_2 + Q_{32} - Q_{20} - Q_{2leak}) \tag{11}$$

where *t* represents time, and h_1, h_2, h_3 represent the liquid levels in each tank; *A* represents the cross section of the tanks; and Q_1, Q_2 denote the flow rates of pumps *1* and *2*; Q_{ij} denotes the flow rates between tank T_i and T_j ($j = 0$ represents the system output) and Q_{ileak} ($i = 1$ or *2* or *3*) represents the output flow of the respective tank when the leak valve is open. These three balance equations make explicit that the volume variance in each tank is equal to the sum of the flow rates that enter and leave the tank. However flows Q_{13}, Q_{32} and Q_{20} are still unknown in equations 9, 10, and 11. To obtain them, the Torricelli's Law is used:

$$Q_{ij} = az_i S_n sgn(h_i - h_j)\sqrt{2g(|h_i - h_j|} \tag{12}$$

where az_i is the outflow coefficient, $sgn(z)$ is the sign of the argument z_i, g is the gravitation constant, and S_n is the cross sectional area of the connecting pipes. So, the resulting equations to calculate the partial flows are:

$$Q_{13} = az_1 S_n sgn(h_1 - h_3)\sqrt{2g|h_1 - h_3|} \tag{13}$$

$$Q_{32} = az_{13} S_n sgn(h_3 - h_2)\sqrt{2g|h_3 - h_2|} \tag{14}$$

$$Q_{20} = az_2 S_n \sqrt{2gh_2} \tag{15}$$

In this paper, our concerned is to maintain the level of the liquid in the tank by manipulating pump liquid flow.

5. CONTROL OF THREE TANK SYSTEM USING NFMPC ALGORITHM

The neuro-fuzzy model predictive controller algorithm (NFMPC) mentioned in the previous section is applied to 3T system by implementing the fuzzy system model as an internal model of the MPC. For the purpose of developing the neuro-fuzzy model a input/output samples data that capture the relationship between the inputs and outputs of the system should be collected. In addition, these data should span across the entire scope of possible variation. The training data were generated by forcing the pump flow with a uniform random signal of 0.27 minimum value and maximum value 0.58 as shown in Figure 3.

The data collected is used for developing a linear autoregressive exogenous (ARX) model and the neuro-fuzzy model. The RBF network model will be used in the nonlinear NFMPC algorithm while the linear ARX model will be used in the linear MPC algorithm, and a comparison between these control algorithms will be considered.

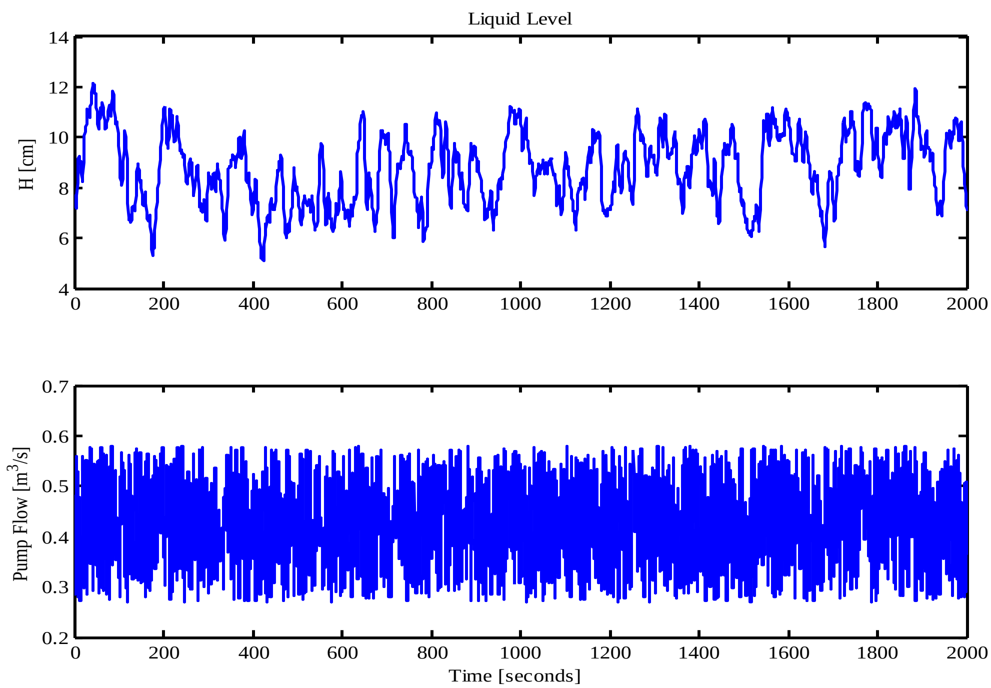


Figure 3. Sample training data

The linear ARX model is identified using the simulated input/output data in the following form equation 16, where the structure of the model is defined by giving the time delay t_d, and the order of the polynomials n_y and n_u, respectively. The structure with parameters $t_d = 2$, $n_y = n_u = 2$ gives the best estimation with 70% validation using the Identification System in the MatLab environment.

$$\begin{aligned} y(t) = f[&y(t-1),\ldots\ldots, y(t-n_y), \\ &u(t-t_d),\ldots\ldots, u(t-t_d-n_u)] \end{aligned} \tag{16}$$

The resulting ARX model has the following form equation 17 without considering the noises model.

$$y(t) = 1.491y(t-1) - 0.5182y(t-2) + 1.417u(t-2) + 0.737u(t-3)$$
(17)

Figure 4 demonstrates the prediction of the identified model by showing how this ARX model can not accurately predict the behavior of the fermentation process, although the overall dynamic characteristics are captured.

Based on the above ARX structure and the input/output data figure 3, a neuro-fuzzy model was identified. Once the neuro-fuzzy model is obtained, some validation tests should be considered. Model validation was performed by application on unseen data Figure 5. The output is generated and compared with the output of the nonlinear simulation with the same inputs. The results of this test shows good agreement between the output of the plant (target data) and neuro-fuzzy (output data).

For the closed-loop simulation, the control algorithms were set up with the neuro-fuzzy network model and the linear ARX model described earlier, and the new set points were introduced.

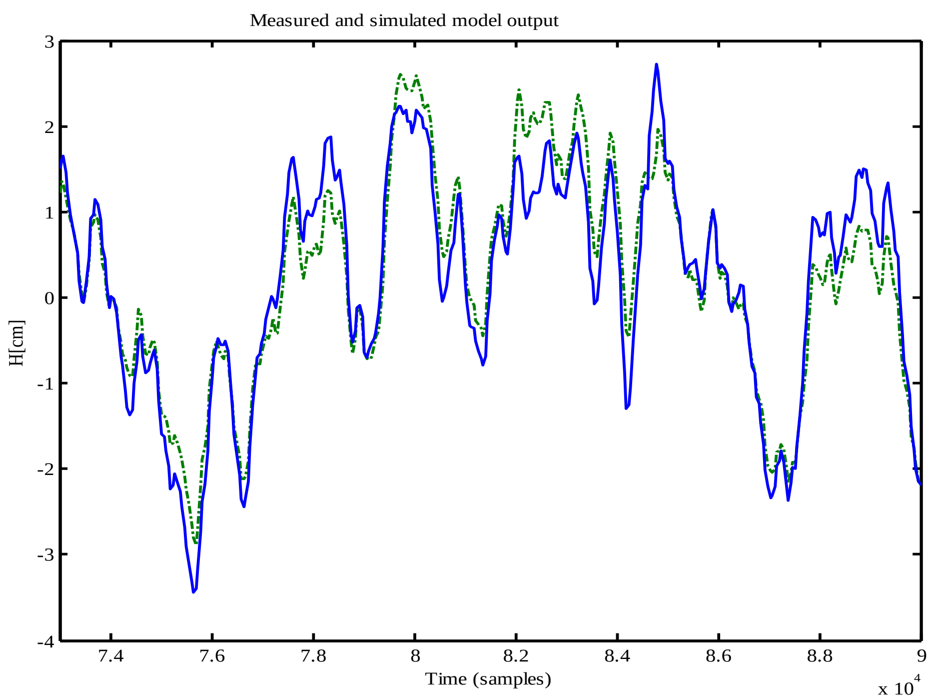


Figure 4. ARX linear model prediction dashed (- - -), measured output solid (—).

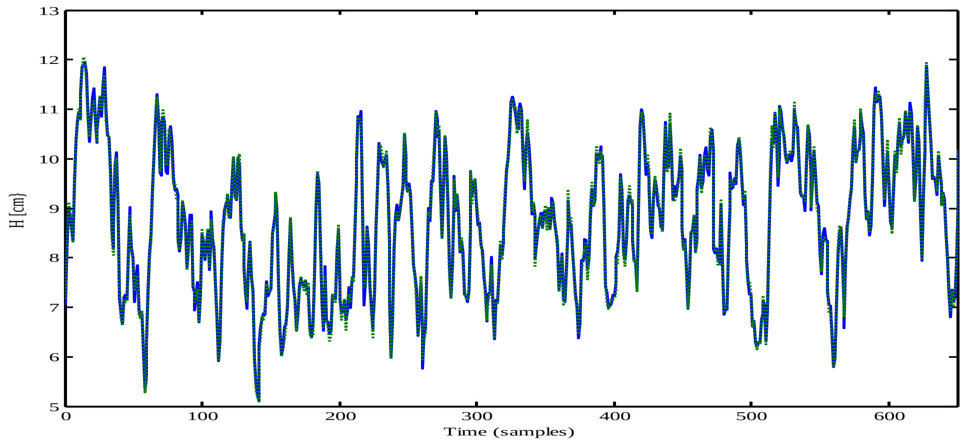
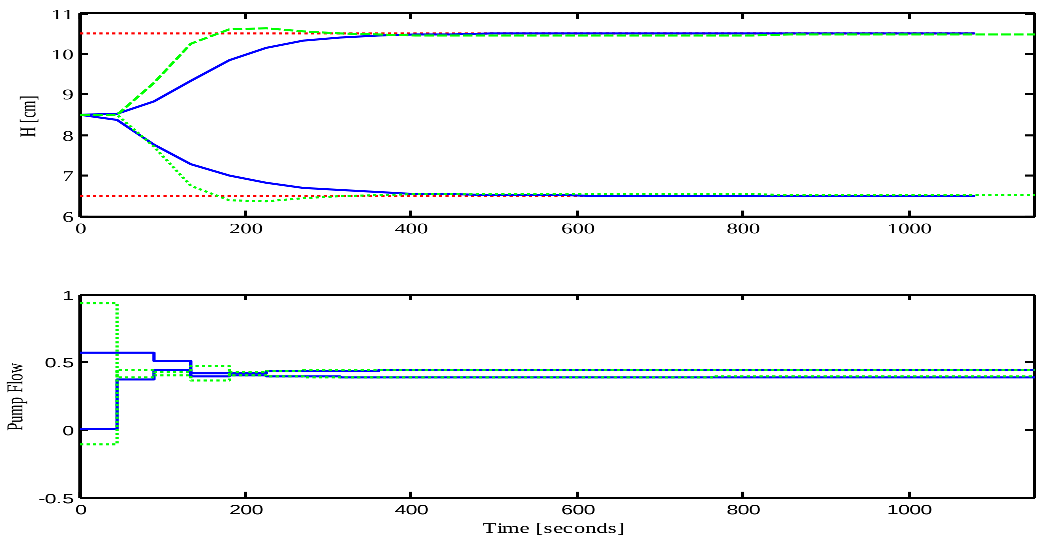


Figure 5. ARX linear model prediction dashed (- - -), measured output solid (—).

The tuning parameters were chosen so that the integrated square error (ISE) between the simulated output and set point is minimized, as p =20, m = 2, Γ^u = 0.95 and $\Gamma^y = 1$. The set point changes were implemented as step changes around nominal values of 0.4158 for the input (Pump flow (m^3/s)), 8.5 for the output (Liquid Level (H cm)), and no filtering was included in the feedback path.

Figure 6 presents the closed loop response of the system for LMPC and NFMPC algorithms, where it can be noticed that the controllers bring the liquid level to the new set points, and they perform the task in a short time. Although, the LMPC gives better output response than NFMPC, but its input response shows a overshoot comparing to the response of the NFMPC. This is because the NFMPC uses an iterative optimization technique which takes long time to process the process a solution than the model inversion technique used in the linear case. Overalls, the NFMPC controller has shown good capability to capture the non-linear dynamics of the fermentation process and respond well to the set point changes.



Key: NFMPC (—), LMPC (- - - -)

Figure 6. Closed-loop response to different set point changes for different algorithms

6. CONCLUSION

This work has presented a model predictive control for controlling a 3T system. To overcome the complexity of the analytical model of this process, the process was modeled using the neuro-fuzzy network from the data generated from the real target, while the other model was developed using the ARX parametric method. The main advantage of the neuro-fuzzy and ARX models compared to the analytical model of the system is that the former design of a controller of a process does not need detailed knowledge about the process, which is a feature that might be of crucial importance in the case of complex processes. The developed models have been used as an internal model of the model predictive control showing good response and ability to capture the dynamic response of the 3T system to set point tracking. It can be conclude that the developed NFMPC controller good performance over the LMPC and would therefore be of significant advantage to the process industry.

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